Name: ______________________________ Date: ____________________

CHAPTER 2: REASONING AND PROOF

Section 2.1 – Inductive Reasoning

Conjecture-

* To show a conjecture is always true, you must __________ __________.

* To show a conjecture is false, you must find a _______________.

Counterexample -

Example #1: Make a conjecture:

A. The sum of two positive numbers is _______________.

B. The product of an even number and an odd number is _______________.

Example #2: Disprove each conjecture by finding a counterexample:

A. For every integer n, n^3 is positive.

B. $5a$ has a value greater than or equal to five for all positive values of a.

Inductive Reasoning – The process of reasoning that a rule or statement is true because specific cases are true.

** Drawing a conclusion from a _______________.

Example #3: Use inductive reasoning to identify the pattern and find the next item in the pattern:

A. January, March, May, _______________.

B. 7, 14, 21, 28, __________.

C. △, □, ⬠, __________.

Name: ______________________________ Date: ______________

CHAPTER 2: REASONING AND PROOF

Section 2.2 – Conditional Statements

Conditional Statement:

- Hypothesis

- Conclusion

Converse:

Inverse:

- negation

Contrapositive:

If today is Monday, then tomorrow is Tuesday.

1. Underline the hypothesis and circle the conclusion.
2. Write the converse. ______________________________
3. Write the inverse. ______________________________
4. Write the contrapositive. ______________________________
5. Is the conditional statement true? Is the converse true?
6. If possible, rewrite as a biconditional. ______________________________

EXAMPLES:

1. Write a counterexample to show that the following statements are false.

   a. If $x^2 = 16$, then x must equal 4.

   b. If a flower is a rose, then it is red.

2. Given the following conditional statement, provide the following.

   If two segments are congruent, then they have the same length.

   a. Converse: ______________________________

   b. Inverse: ______________________________

3. Given the following conditional statement, provide the following.

   If there is snow on the ground, then the flowers will not bloom.

   a. Converse: ______________________________

   b. Inverse: ______________________________

   c. Contrapositive: ______________________________

Name: ______________________________ Date: ______________

DEDUCTIVE REASONING

DEDUCTIVE REASONING: *Coming up with a conclusion based on statements that are assumed to be true.*

1. Marty bowled from 7:00 to 10:00 on Wednesday.
   At 8:00 on Wednesday evening, Marty's car was seen running a red light.

   WHAT CAN YOU DEDUCE?

2. Mrs. Jones came home at 4:30 p.m. to find the cookie jar empty although she had filled it that morning.
   Her husband came home for lunch.
   Mrs. Jones's daughter said the cookie jar was empty when she came home after school at 3 p.m.
   Mrs. Jones's son came home at 4 p.m.

   WHAT CAN YOU DEDUCE?

3. The measures of the angles of a triangle add up to 180°.
   One angle of the triangle measures 90°.

   WHAT CAN YOU DEDUCE?

4. The measures of the angles of a triangle add up to 180°.
   One angle of the triangle measures 90°.
   The other two angles have the same measure.

   WHAT CAN YOU DEDUCE?

5. Given that all widgets are doodads and all snickets are widgets, which of the following must be true?

   (a.) All widgets are snickets. (b.) All doodads are snickets.
   (c.) All doodads are widgets. (d.) All snickets are doodads.

A **THEOREM** is a statement that can be proven by other statements that we assume to be true.

6. Reorganize the statements below so that they are easier to follow, and so that it proves the theorem below. Rewrite the statements on another sheet of paper. (Be careful, one statement can be a bit tricky)

THEOREM: If there is no Great Pumpkin, then Snoopy won't have pie for dinner.

PROOF:

If Lucy plays a trick on Charlie Brown, he will be upset.
If Linus is mistaken, Lucy is pleased.
If Lucy becomes unruly, she plays a trick on Charlie Brown.
If there is no Great Pumpkin, then Linus is mistaken.
If Charlie Brown forgets to feed Snoopy, Snoopy won't have pie for dinner.
If Lucy is pleased, she becomes unruly.
Charlie Brown forgets to feed Snoopy if he is upset.

7. Rewrite the following statements in "if-then" form. Then, on another sheet of paper, rearrange them in a logical order.

NASA would send some mice on a lunar mission if they were eager astronauts.
If mice were sent on a lunar mission, the eyes of the entire world would be watching them on television.
If the moon were made of green cheese, mice would make eager astronauts.
It would be one giant peep for mousekind if the entire world were watching them on television.

8. What "theorem" is proved by the four statements in problem 7?

9. Using problems 6 and 6 as a model, create a "Theorem" and "mixed-up proof" of your own.

- It must have a minimum of 7 lines.
- It must have a maximum of 15 lines.
- You must turn in one page with the Theorem and the correct proof.
- You must turn in one page with the Theorem and the mixed-up proof.
- All pages must be typed.
- Be creative.
- Have fun!

If you think about being a chef, you will sign up for classes at the Tech School.

If your grades improve, your friends and family will be proud of you.

If you have free time, you'll get bored.

If you are happy, then you'll make a cake.

If you do your homework, then your grades will improve.

You'll sleep through you're favorite t. v. show, if you fall asleep.

You'll only have to take four classes at the high school, if you sign up for classes at the Tech School.

You'll share the cake with your friends, if you make a cake.

You'll be happy, if your friends and family are proud of you.

If your receive compliments on your baking, then you consider becoming a chef.

You'll fall asleep, if you're bored.

You do your homework, if you are responsible.

You will have a lot of free time on your hands, if you only have to take four classes.

If you share with your friends, they will compliment you on being such a great baker.

Name: ______________________________ Date: ____________________

CHAPTER 2: REASONING AND PROOF

Section 2.4 – Biconditional Statements

When you combine a ____________ ____________ and its ____________, you create a ____________ ____________ .

A biconditional statement is written in the form: " ***p* if and only if *q*** "

$$p \leftrightarrow q \text{ means } p \rightarrow q \text{ and } q \rightarrow p$$

In geometry, ______________ are statements that can be written as a true biconditional.

*The biconditional statement is only true if the conditional and converse statements are **both** true!!

Biconditional Statement: "Two angles are congruent if and only if their measures are equal."

Conditional Statement: ________________________________

Converse Statement: ________________________________

Biconditional Statement: " Points are collinear if and only if they lie on the same line. "

Conditional Statement ________________________________

Converse Statement ________________________________

Biconditional Statement: ________________________________

Conditional Statement " If 2x + 5 = 11, then x = 3 "

Converse Statement ________________________________

Determine if each biconditional statement is true. If false, give a counterexample.

1. An angle is a right angle iff its measure is 90°.

2. $y = -5 \leftrightarrow y^2 = 25$

3. a = 4 and b = 3 iff ab = 12.

Write each definition as a biconditional statement.

4. An isosceles triangle has at least two congruent sides.

5. The measure of a straight angle in 180°.

6. A quadrilateral is a four-sided polygon.

CW / HW : pg.99 #1-23

The Big List of Properties

Addition Properties	Examples
If a segment is added to two congruent segments, then the sums are congruent. If an angle is added to two congruent angles, then the sums are congruent.	
If congruent segments are added to congruent segments, then the sums are congruent. If congruent angles are added to congruent angles, then the sums are congruent.	

Subtraction Properties	
If a segment (or angle) is subtracted from congruent segments (or angles), then the differences are congruent.	
If congruent segments (or angles) are subtracted from congruent segments (or angles), then the differences are congruent.	

Multiplication Property

If segments (or angles) are congruent, then their like multiples are congruent.	

Division Property

If segments (or angles) are congruent, then their like divisions are congruent.	

Transitive Properties

If angles (or segments) are congruent to the same angle (or segment), then they are congruent to each other.	
If angles (or segments) are congruent to congruent angles (or segments), then they are congruent to each other.	

Substitution Property

Reflexive Property

Symmetric Property

Name ______________________ Date __________

For Exercises 1–12, write the letter of each property next to its definition. The letters *a*, *b*, and *c* represent real numbers.

1. If $a = b$, then $b = a$. ________
2. If $a = b$, then $ac = bc$. ________
3. $\overline{AB} \cong \overline{AB}$ ________
4. $a = a$ ________
5. If $a = b$, then $a + c = b + c$. ________
6. $a(b + c) = ab + ac$ ________
7. If $a = b$ and $b = c$, then $a = c$. ________
8. If $\angle P \cong \angle Q$, then $\angle Q \cong \angle P$. ________
9. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. ________
10. If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. ________
11. If $a = b$, then b can be substituted for a in any expression. ________
12. If $a = b$, then $a - c = b - c$. ________

A. Addition Property of Equality
B. Subtraction Property of Equality
C. Multiplication Property of Equality
D. Division Property of Equality
E. Reflexive Property of Equality
F. Symmetric Property of Equality
G. Transitive Property of Equality
H. Substitution Property of Equality
I. Distributive Property
J. Reflexive Property of Congruence
K. Symmetric Property of Congruence
L. Transitive Property of Congruence

Identify the property that justifies each statement.

13. $m = n$, so $n = m$.

____

14. $\angle ABC \cong \angle ABC$

____

15. $\overline{KL} \cong \overline{LK}$

____

16. $p = q$ and $q = -1$, so $p = -1$.

____

17. If $\angle ABC \cong \angle DEF$, then $\angle DEF \cong \angle ABC$.

____

18. $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, so $\angle 1 \cong \angle 3$.

____

19. If $FG = HJ$, then $HJ = FG$.

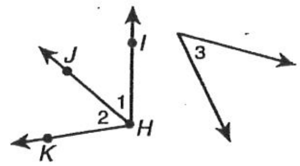
____

20. $\overline{WX} \cong \overline{WX}$

____

Name ______________________________ Date ____________

I. **Write the letter of the correct justification next to each step. (Use one justification twice.)**



Given: $\overrightarrow{HJ}$ is the bisector of $\angle IHK$ and $\angle 1 \cong \angle 3$.

1. $\overrightarrow{HJ}$ is the bisector of $\angle IHK$. ________

2. $\angle 2 \cong \angle 1$ ________

3. $\angle 1 \cong \angle 3$ ________

4. $\angle 2 \cong \angle 3$ ________

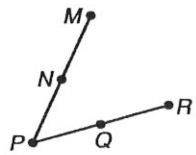
A. Definition of $\angle$ bisector

B. Given

C. Transitive Prop. of $\cong$

II. **Given:** N is the midpoint of $\overline{MP}$, Q is the midpoint of $\overline{RP}$, and $\overline{PQ} \cong \overline{NM}$.

Prove: $\overline{PN} \cong \overline{QR}$



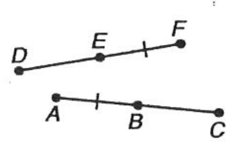
Write a justification for each step.

Proof:

1. N is the midpoint of $\overline{MP}$.	1. ________________
2. Q is the midpoint of $\overline{RP}$.	2. ________________
3. $\overline{PN} \cong \overline{NM}$	3. ________________
4. $\overline{PQ} \cong \overline{NM}$	4. ________________
5. $\overline{PN} \cong \overline{PQ}$	5. ________________
6. $\overline{PQ} \cong \overline{QR}$	6. ________________
7. $\overline{PN} \cong \overline{QR}$	7. ________________

III **Write a justification for each step.**

Given: $AB = EF$, B is the midpoint of $\overline{AC}$, and E is the midpoint of $\overline{DF}$.



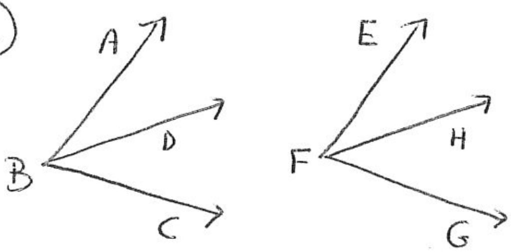
1. B is the midpoint of $\overline{AC}$, and E is the midpoint of $\overline{DF}$.	________________
2. $\overline{AB} \cong \overline{BC}$, and $\overline{DE} \cong \overline{EF}$.	________________
3. $AB = BC$, and $DE = EF$.	________________
4. $AB + BC = AC$, and $DE + EF = DF$.	________________
5. $2AB = AC$, and $2EF = DF$.	________________
6. $AB = EF$	________________
7. $2AB = 2EF$	________________
8. $AC = DF$	________________
9. $\overline{AC} \cong \overline{DF}$	________________

NAME: ______________________________ DATE: ________________

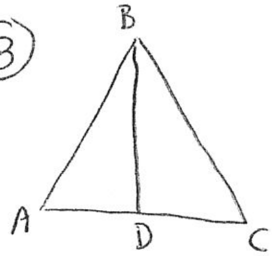
Directions: Identify the rule used to come up w/each conclusion.

1) 

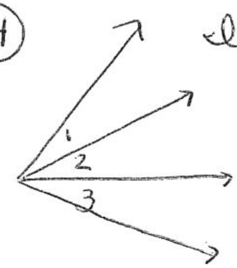
If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

2) 

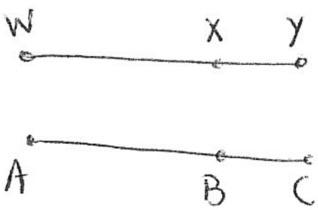
If $\angle ABC \cong \angle EFG$, $\overrightarrow{BD}$ bisects $\angle ABC$, and $\overrightarrow{FH}$ bisects $\angle EFG$, then $\angle ABD \cong \angle EFH$.

3) 

$\overline{BD} \cong \overline{BD}$

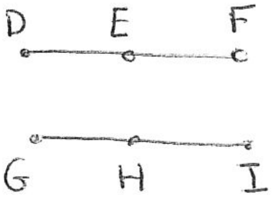
4) 

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

5) 

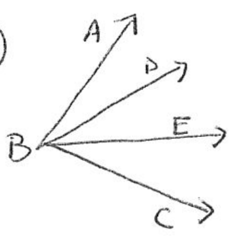
If $\overline{WY} \cong \overline{AC}$ and $\overline{WX} \cong \overline{AB}$, then $\overline{XY} \cong \overline{BC}$.

Directions: Complete each statement using the rule stated.

6) 

If $\overline{DE} \cong \overline{GH}$, E midpt of DF, & H midpt of $\overline{GI}$, then ________________

(Multiplication Property)

7) 

If $\angle ABE \cong \angle DBC$, then ________________

(Subtraction Property).

Name: ______________________________ Date: ________________

UNDERSTANDING PROOFS

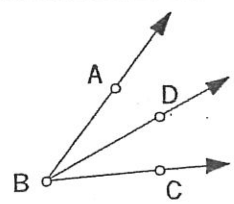
* A proof is a series of sequential steps of justified conclusions.
The conclusions are justified by either a definition, postulate, property, or previously proved theorem.

*After you read the given information and what you are to prove, you must figure out what information applies (what definitions, postulates, properties, or theorems might help you).

EXAMPLE:

Given: $\overrightarrow{BD}$ bisects $\angle ABC$.
$m\angle ABD = 30°$

Prove: $m\angle DBC = 30°$



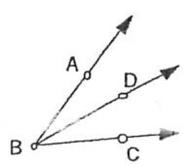
Possible applicable information: bisect and congruent

statements	reasons
1) $\overrightarrow{BD}$ bisects $\angle ABC$ $m\angle ABD = 30°$	1) given
2) $m\angle ABD = m\angle DBC$	2) Def. $\angle$ bisect
3) $30° = m\angle DBC$	3) substitution - 1, 2
4) $m\angle DBC = 30°$	4) symmetric

Proofs For You To Try!

Before you begin writing the proof, first sketch the picture if necessary. Second, in the right hand column, write all the definitions, postulates, and other rules that might be helpful for you to write this proof.

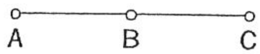
Proof	**Helpful Information**
1) Given: $m\angle ABD = 30°$ $m\angle DBC = 30°$ Prove: $\overrightarrow{BD}$ is the angle bisector	



Proof	Helpful Information

2) Given: B is the midpoint of $\overline{AC}$
AB = 10

Prove: CB = 10

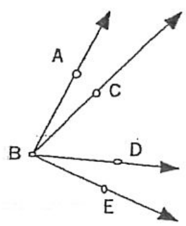


3) Given: $\angle 1$ and $\angle 2$ are right $\angle$s

Prove: $\angle 1 \cong \angle 2$

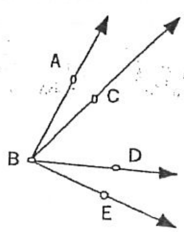
4) Given: $\angle ABC \cong \angle DBE$

Prove: $\angle ABD \cong \angle CBE$



5) Given: $m\angle ABD = 60°$
$m\angle EBC = 60°$

Prove: $\angle ABC \cong \angle EBD$



Name: ______________________________ Date: ________________

A FEW MORE RULES YOU MAY USE AS REASONS IN YOUR PROOFS . . .

PERPENDICULAR:

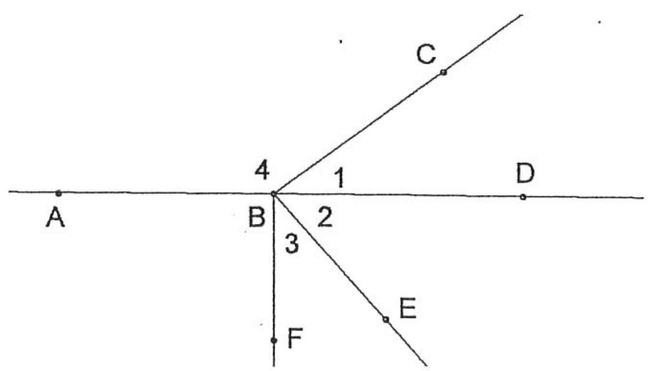
RIGHT ANGLE CONGRUENCY THEOREM:

CONGRUENT COMPLEMENTS THEOREM:

CONGRUENT SUPPLEMENTS THEOREM:

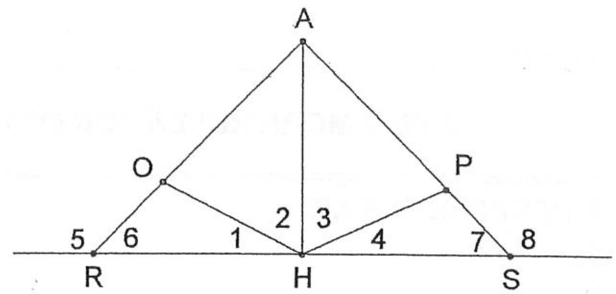
1. GIVEN: $\overrightarrow{BC} \perp \overrightarrow{BE}$ and $\overrightarrow{BD} \perp \overrightarrow{BF}$

PROVE: $\angle 1 \cong \angle 3$



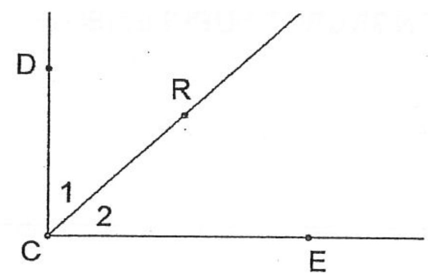
2. GIVEN: $\angle 6 \cong \angle 7$

PROVE: $\angle 5 \cong \angle 8$



3. GIVEN: $\overrightarrow{CD} \perp \overrightarrow{CE}$

PROVE: $\angle 1$ and $\angle 2$ are complementary



Name: ______________________________ Date: ______________

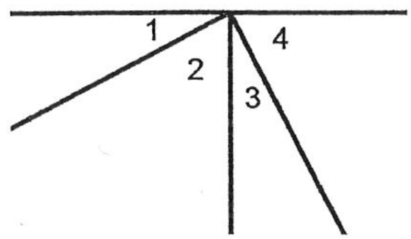
MORE PRACTICE WITH WRITING PROOFS

1. GIVEN: $\overline{AB} \cong \overline{CD}$ and AC = 12

PROVE; BD = 12

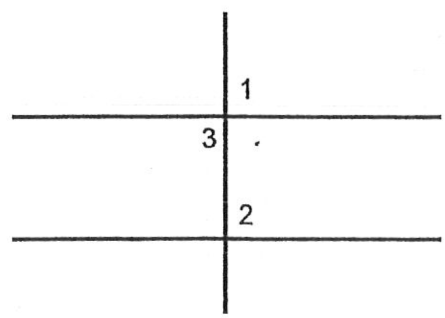
2. GIVEN: $\angle 1$ and $\angle 2$ are complementary
$\angle 1 \cong \angle 3$
$\angle 2 \cong \angle 4$

PROVE: $\angle 3$ and $\angle 4$ are complementary



3. GIVEN: $\angle 1 \cong \angle 2$ and $m\angle 3 = 90°$

PROVE: $\angle 2$ is a right angle

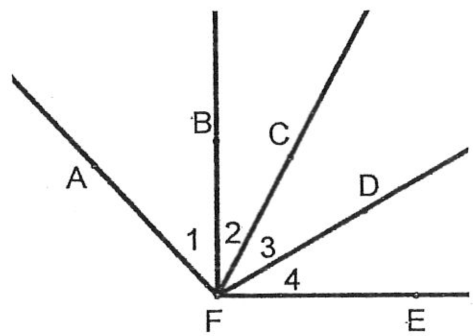


4. GIVEN: $\angle 1$ and $\angle 2$ are vertical angles
$\angle 1$ and $\angle 3$ are linear pairs

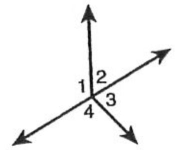
PROVE: $\angle 2$ and $\angle 3$ are supplementary

5. GIVEN: $\angle AFC \cong \angle EFC$ and $\angle 2 \cong \angle 3$

PROVE: $\angle 1 \cong \angle 4$

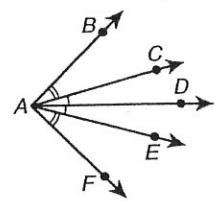


Name ______________________ Date ________



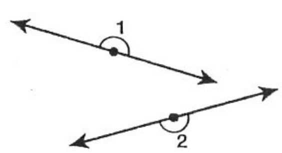
1. **Given:** $\angle 1$ and $\angle 2$ form a linear pair, and $\angle 3$ and $\angle 4$ form a linear pair.
Prove: $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$

Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair, and $\angle 3$ and $\angle 4$ form a linear pair.	1. **a.** ______________
2. $\angle 1$ and $\angle 2$ are supplementary, and $\angle 3$ and $\angle 4$ are supplementary.	2. **b.** ______________
3. **c.** ______________ ______________	3. Def. of supp. $\angle$s
4. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$	4. **d.** ______________



2. **Given:** $m\angle BAC = m\angle EAF$, $m\angle CAD = m\angle DAE$
Prove: $m\angle BAD = m\angle DAF$

Statements	Reasons
1. $m\angle BAC = m\angle EAF$, $m\angle CAD = m\angle DAE$	1. **a.** ______________
2. **b.** ______________ ______________	2. Add. Prop. of =
3. $m\angle BAC + m\angle CAD = m\angle BAD$, $m\angle EAF + m\angle DAE = m\angle DAF$	3. $\angle$ Add. Post.
4. $m\angle BAD = m\angle DAF$	4. **c.** ______________



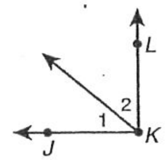
3. **Given:** $\angle 1$ and $\angle 2$ are straight angles.
Prove: $\angle 1 \cong \angle 2$
Proof:

Statements	Reasons
1. **a.** ______________	1. Given
2. $m\angle 1 = 180°$, $m\angle 2 = 180°$	2. **b.** ______________
3. $m\angle 1 = m\angle 2$	3. Subst. Prop. of =
4. **c.** ______________	4. Def. of $\cong$ $\angle$s

Name ______________________________ Date __________

1. **Given:** $\angle JKL$ is a right angle.

Prove: $\angle 1$ and $\angle 2$ are complementary angles.

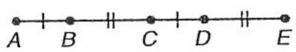


Two-Column Proof:

Statements	Reasons
1. $\angle JKL$ is a right angle.	1. Given
2. $m\angle JKL = 90°$	2.
3. $m\angle JKL = m\angle 1 + m\angle 2$	3.
4. $90° = m\angle 1 + m\angle 2$	4.
5. $\angle 1$ and $\angle 2$ are complementary angles.	5.

2. **Given:** $AB = CD$, $BC = DE$

Prove: C is the midpoint of $\overline{AE}$.

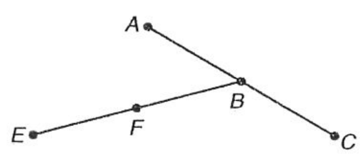


Statements	Reasons
1. $AB = CD$, $BC = DE$	1. Given
2. $AB + BC = CD + DE$	2.
3. $AB + BC = AC$, $CD + DE = CE$	3.
4. $AC = CE$	4. Subst. (,)
5. $\overline{AC} \cong \overline{CE}$	5.
6. C is the midpoint of $\overline{AE}$.	6.

3.

Given: $\overline{EF} \cong \overline{BC}$, B is the midpoint of $\overline{AC}$.

Prove: $\overline{EF} \cong \overline{AB}$



CHAPTER 2 REVIEW

Terminology, Rules, and Other Things You Should Know

Conditional Statement
Hypothesis
Conclusion
Converse
Inverse
Contrapositive
Biconditional

Deductive Reasoning
Inductive Resoning
Counterexample
Logical Chains
Addition Property
Subtraction Property
Multiplication Property

Division Property
Reflexive Property
Symmetric Property
Transitive Property
Substitution
Perpendicular
Right angle ≅ Theorem

≅ Complements Theorem
≅ Supplements Theorem
How to write a proof
Theorem/Postulate

You should still know EVERYTHING from Chapter 1

Format of the Test

Part 1 - Worth 47 points
28 Questions
Multiple Choice
Identifying Conditional Stmts.
Inductive Reasoning
Deductive Reasoning
Proving/Disproving Conjectures
Math Problems involving angles
Always, Sometimes, or Never

Part 2 - Worth 39 points
16 Questions
Multiple Choice
Biconditionals
Matching (Properties/Rules)
3 Proofs

PRACTICE PROBLEMS → The best problems that I can find are the ones that we've already done. If you're planning on studying (which I hope you are), redo the following worksheets:

Sections 2.1 to 2.4 Quiz, Homework Worksheet on the Properties (this is hand-written),
3 Proof Worksheets (any proof 5 steps or less is fair game)

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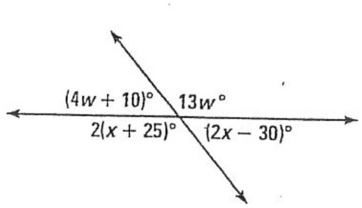
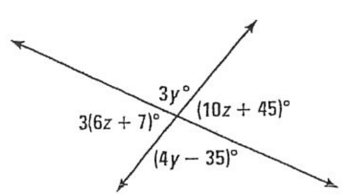
1. Solve for w and x.

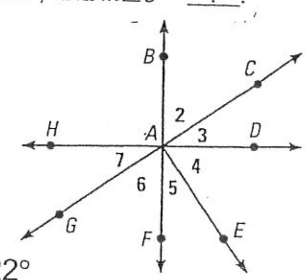


2. Solve for y and z.



3. $\overleftrightarrow{BF} \perp \overleftrightarrow{HD}$ and $\overleftrightarrow{GC} \perp \overrightarrow{AE}$.

a. If $m\angle 3 = 31°$, then $m\angle 5 =$ _?_.
b. If $m\angle 5 = 29°$, then $m\angle 4 =$ _?_.
c. If $m\angle CAF = 122°$, then $m\angle GAB =$ _?_.
d. If $m\angle 7 = 35°$, then $m\angle 3 =$ _?_.



1. w = 10
x = 40

2. y = 35
z = 3

3a. 31°
3b. 61°

3c. 122°
3d. 35°